Taylor Method

Initial Value Problem to be solved: $y'(x) = f(x, y)$, $y(x_0) = y_0$

Let (x,y) be either the initial point or a point which has already been computed.

Change in x: $\Delta x = h$ (given)

Recall the Taylor Expansion from Calculus:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$

In the n^{th} – order Taylor method, the change in y is given by

$$\Delta y = hy' + \left(\frac{h^2}{2!}\right)y^{(2)} + \left(\frac{h^3}{3!}\right)y^{(3)} + \ldots + \left(\frac{h^n}{n!}\right)y^{(n)}$$

If n = 1, we have Euler's Method (ie: $\Delta y = hy'$).

If n ≥ 2, we calculate the formulas for $y^{(2)}$, $y^{(3)}$, … , $y^{(n)}$ by differentiating the given differential equation. Then $y^{(1)}$, $y^{(2)}$, … , $y^{(n)}$ are all evaluated at the known (x,y) point.

New x = x + h

New y = y + Δy

This gives the new (x,y) point. We continue in the same way for the next point.

In the n^{th} – order Taylor Method, it is assumed that f has continuous partial derivatives of all orders up to and including the n^{th} order.

Note: the x – increment (ie: h) can be negative.